Характеристики квазилинейного ур-я

В акуст. приближ-ии была система

$$\begin{cases} \frac{\partial \rho}{\partial t} + \rho_0 \frac{\partial v}{\partial x} = 0 \\ \frac{\partial v}{\partial t} + \frac{c_0^2}{\rho_0}\frac{\partial \rho}{\partial x} = 0 \end{cases}$$

$a(x,t)\frac{\partial u}{\partial t} + b(x,t)\frac{\partial u}{\partial x} = 0$; Производную u по l (произв. по напр.) $\frac{du}{dl} = (\text{grad} u, l) = (\text{grad} u, \{dx, dy\}) =$

$= \left(\frac{\partial u}{\partial t}\frac{dt}{d\tau} + \frac{\partial u}{\partial x}\frac{dx}{d\tau}\right)d\tau$; $\frac{du}{dl} = 0 \Rightarrow \frac{\partial u}{\partial t}\frac{dt}{d\tau} + \frac{\partial u}{\partial t}\frac{dx}{d\tau} = 0$

$\frac{dt}{d\tau} = a(x,t)$; $\frac{dx}{d\tau} = b(x,t)$; $\frac{dx}{dt} = \frac{dx}{d\tau}\cdot\frac{1}{\frac{dt}{d\tau}} = \frac{b}{a}$; $\frac{\partial u}{\partial t} + \frac{b}{a}(x,t)\frac{\partial u}{\partial x} = 0$; $l: \frac{dx}{dt} = \frac{b}{a}$

напр-е, вдоль которого u сохр-ет своё зн-е

Характеристики и характ-ая форма записи системы квазилин-х ур-ий.

$C(x,t,\vec{u})\frac{\partial \vec{u}}{\partial t} + B(x,t,\vec{u})\frac{\partial \vec{u}}{\partial x} = \vec{f} \Rightarrow \frac{\partial \vec{u}}{\partial t} + A(x,t,\vec{u})\frac{\partial \vec{u}}{\partial x} = \vec{b}$; $A = C^{-1}B$; $\vec{b} = C^{-1}f$

$\vec{l}A = \vec{l}\lambda$; $l^k\frac{\partial u}{\partial t} + l^k A\frac{\partial u}{\partial x} = (l^k, b) \Rightarrow l^k\frac{\partial u}{\partial t} + l^k\lambda_k\frac{\partial u}{\partial x} = (l^k, b)$

$$\sum_{i=1}^{n} l_i^k \underbrace{\left(\frac{\partial u_i}{\partial t} + \lambda_k\frac{\partial u_i}{\partial x}\right)} = \sum_{i=1}^{n} l_i b_i$$

характ. форма с.к.у.

Инварианты Римана для СКУ (для гипербол. систем)

$$\frac{\partial \vec{u}}{\partial t} + A(x,t,\vec{u})\frac{\partial \vec{u}}{\partial x} = \vec{b}$$

$$\sum_{i=1}^{n} l_i^k\left(\frac{\partial u_i}{\partial t} + \lambda^k\frac{\partial u_i}{\partial x}\right) = \sum_{i=1}^{n} l_i b_i \quad k = 1, \dots, n$$

$r_k = (l^k, \vec{u}) = \sum_{i=1}^{n} l_i^k u_i$ - пер-е (инвар-ты Римана)

$\vec{r} = \lambda\vec{u}$

$$\left(\frac{d}{dt}\underbrace{l^k\vec{u}}_{r^k}\right)_k = \left(l^k, \frac{d\vec{u}}{dt}\right) + \left(\vec{u}\frac{dl^k}{dt}\right) = (l^k, \vec{b}) + \vec{u}\frac{dl^k}{dt}$$

произв-я по напр-ю $\quad \vec{u} = \lambda^{-1}\vec{r}$

$$\left(\frac{d}{dt}\vec{r}\right)_k = (\vec{l}^k, \vec{b}) + \left(\Lambda^{-1}\vec{r}, \frac{d\vec{l}^k}{dt}\right)$$

↑ запись системы в инв-ах Римана

$A(x,t) \Rightarrow l(x,t), \; \lambda(\vec{r}, t)$

$$l^k \frac{\partial u}{\partial t} + \lambda_k \frac{\partial u}{\partial x} = b$$

$$\frac{\partial}{\partial t}(r^k) + \lambda_k \frac{\partial (r_k)}{\partial x} = f^k + \vec{u}\,\frac{d\vec{l}^k}{dt} + \lambda_k \vec{u}\,\frac{\partial \vec{l}^k}{\partial x} \Rightarrow$$

$$\Rightarrow \frac{\partial r^k}{\partial t} + \lambda_k \frac{\partial r^k}{\partial x} = f^k + \Lambda^{-1} r \left(\frac{dl}{dt}\right)_k$$

система в инв-ах Римана